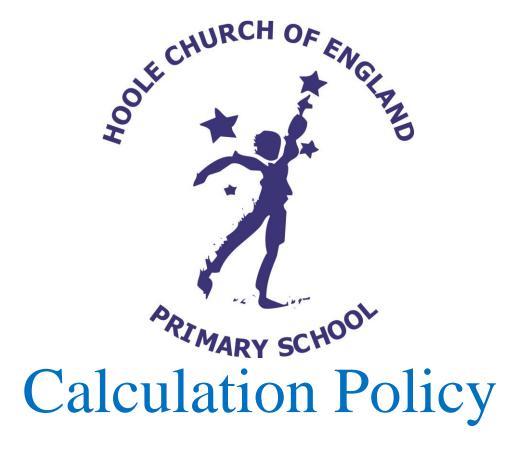
# Hoole Church of England Primary School



# Addition

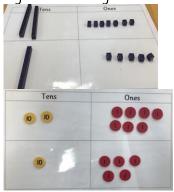


#### Concrete

Children in year 3 begin adding 2 digit numbers where exchanging isn't needed using base 10 as a review from previous year. They then progress to doing this with the place

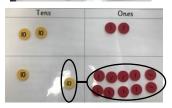


When this has been mastered children will need to add with numbers that need exchanging, using base 10. They will then begin to exchange with the place value counters.

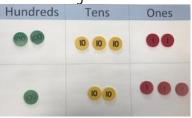


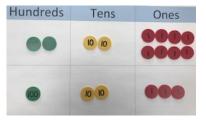
value counters.





The children should then progress to 3 digit numbers following the same teaching process from before: adding with no exchanges, adding where the ones exchange, adding where the tens exchange and then both.



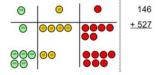


# **Pictorial**

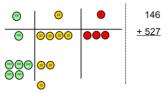
The children will begin by recapping the part-whole model from Year 2.



Buiulding on this, the children can then progress to drawing a pictoral representation of the columns and place value counters. First of all alongside the practical equipment and then without.



Add up the ones and exchange 10 ones for one 10. The children can draw an arrow underneath to identify which place value column they move into.



Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

Children should use the pictorial representations alongside the abstract.

# Abstract

Building on from the numberline in Year 2, Year 3 children will start adding using their knowledge of partitioning.

They will then partition 2 digit numbers into tens and ones. They will add the tens, add the ones and then combine the answer together.

$$25 + 13 = 20 + 10 = 30$$
 $5 + 3 = 8$ 
 $30 + 8 = 38$ 
 $20 + 5$ 
 $40 + 8$ 
 $60 + 13 = 73$ 

Here are 2 possible ways of recording this method.

Once children are confident with this, they will move on to recording this using the expaned method. They should begin with numbers that don't require exchanging, building up to the ones exchanging and then the tens.

536

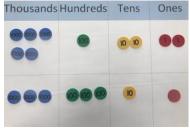
<u>+ 85</u>

621 11

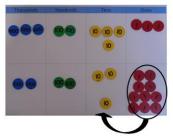
They should then be confident to use this method using 3 digit numbers. Some children may even be confident with using the formal column method of addition if they have mastered palce value and exchanging.

# Concrete

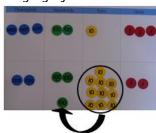
Children begin Year 4 re-visiting learning from Year 3. They should then move on to adding with 4 digit numbers, with no exchanging.



Once they have grasped this they can begin working with numbers that require exchanging. First of all exchanging the ones then the tens and finally the hundreds and thousands.



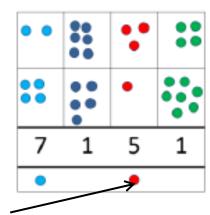
This shows the exchanging of 10 ones into 1 ten.



This shows the exchanging of 10 tens into 1 hundred.

# Pictorial

Children should then be encouraged to draw the place value counters, keeping them in their place value columns.



They need to ensure that they record any place value counters that they have exchanged.

# Abstract

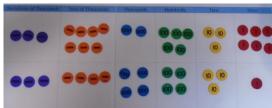
Again, children will begin by adding using the expanded method to ensure they have the understanding of the place value of the numbers that they are adding.

They should move on to using this method with numbers that require exchanging. First beginning with ones and then progressing to the thousands.

The children should then be ready for the formal column method of addition with 4 digits.

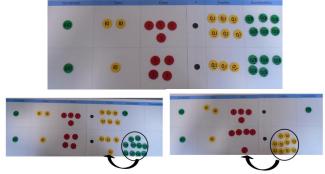
## Concrete

In Year 5, children will continue to build on the knowledge of addition learnt in previous years. They will be required to add 6 digit numbers. Children can begin by exploring adding these with place value counters.

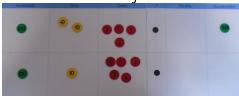


It is clear from this that the children will need to exchange 10 ten thousands for 1 hundred thousand.

As children move on to decimals, money and decimal place value counters can be used to support learning.



This shows the children exchanging 10 hundredths for 1 tenth and then 10 tenths for 1 whole one.



They should then be able to state that the answer is 239.01.

# Pictorial

To consolidate the learning of the place value columns of 6 digit numbers, children could begin to represent the counters in a pictorial appraoch. However, this will be a time consuming method and this should be explored with the children.

The children should be using bar model representations to solve multi-step word problems.

e.g
In Hoole school library there are 2671 books. There are 356 poetry books and 769 non-fiction books. How many fiction books are there?

2671		
356	769	?

# Abstract

Children will use column addition to add numbers up to 6 digits. They should be confident with this method for the beginning of the year.

When covering decimalss, children will be expected to add and subtract decimal numbers through money and measure. They need to ensure that the decimal point is kept in line and should also be written in the answer, ensuring that the tenths and hundredths are in the correct place value columns.

Empty decimal places should be filled with a place holder and children should be using the correct terminology when adding (seen here in red).

They should be able to add more than 2 amounts together confidently.

	Year 6	
Concrete	Pictorial	Abstract
Children can begin to add 7 digit numbers together using practical resources. However, they should now be fully skilled to transfer this straight into the abstract representation.	Again, children can draw the place value counters in the correct place value columns to help them with adding.	Children should use column addition to add 7 digit numbers confidently, exchanging numbers that are larger than 1.
When adding decimals, the children can begin with place value counters to build on knowledge from Year 5 and thousandths should be introduced.  Create the numbers.  This shows the process of exchanging 10 thousandths for 1 hundredth.  This then shows the process of exchanging 10 tenths for 1 whole one.  They should be aware that the answer is 654.25 as there is no	The children will use the bar model to represent complicated word problems including addition.  e.g England recruited 82,016 soldiers, trained 5,399 nurses and 14, 375 pilots. What is the total amount of people?  ? 5,399 14,375 82,016	7630349 $7821785$ $+908678$ $16360812$ $2$
need to place a 0 where there is nothing in the thousandths column.		Children should now have mastered exchanging and should be able to do this confidently multiple

times within a calculation.

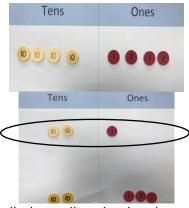
# Subtraction



# Concrete

Children in Year 3 will begin by using Base 10 to make the bigger number to consolidate learning from Year 2. They will then take the smaller amount away and record what is left.

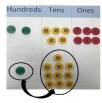
The children will then progress to using place value counters. They should begin with subtracting where no exchanging is needed.



The children will physically take the place value counters away to show how much is left. They should then progress to exchanging so that they exchange 1 ten into ten ones.

Children should then be confident calculating with 3 digit numbers following the same process.



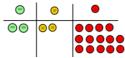


This shows exchanging 10 ones into 1 ten and ten tens into 1 hundred.

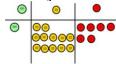
# **Pictorial**

Once children are confident with this using practical resources, they can begin to draw the representations alongside. The should cross out the counters of the amount that they are taking away.

There are not enough counters to take away 8 ones. They will need to exchange 1 ten for 10 ones and then cross out 8 ones.



Now there are not enough counters in the tens column to take away 8 tens. They will need to exchange 1 hundred for 10 tens.



Then they can cross out 8 tens.

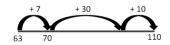
The children should then progress to recording the calculation next to their diagrams.

To help them to understand the operation needed in the question, children will use the bar model to represent the question.

178	
48	?

# Abstract

In Year 3, children will begin by finding the difference and counting on using a number line.



10 + 30 + 7 = 47

The children will begin by recording using the expanded method, beginning with 2 digit numbers and progressing to three digits.

$$267 - 131 =$$

$$200 + 60 + 7$$

$$- 100 + 30 + 1$$

$$100 + 30 + 6$$

Once the children are secure with the 'exchanging' using practical equipment and pictorial representations, they can use this to subtract two and three-digit numbers in a variety of contexts.

$$363-178 = 200^{1}50$$

$$380 + 60 + 13$$

$$-100 + 70 + 8$$

$$100 + 80 + 5$$

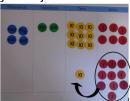
If children are ready, and have a secure understanding of the maths involved, they may use compact column subtraction for three-digit numbers. However, children must not be moved onto this stage too soon.

	<sup>2 15</sup> <sub>1</sub> <b>3 6</b> 3
-	178
	185

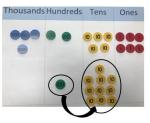
### Concrete

Children will begin with subtracting 3 digit numbers, recapping from previous year.

After this, they will progress to 4 digit numbers. Make the larger number with the place value counters. Starting with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones.



Now I can subtract 8 ones. Looking at the tens, can I take away 8 tens easily? I need to exchange one hundred for  $10 \text{ tens } (10 \times 10)$ .



Now I can take away eight tens and subtract my hundreds, followed by the thousands.

Practical resources need to be used when there is a 0 in one of the place value columns.







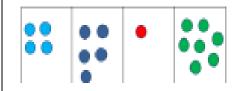


This shows that the children would need to exchange from the hundreds column and then the tens column.

## Pictorial

Show children how the concrete method links to the written method alongside their working. Cross out the numbers when exchanging and show where we write our new amount.

4517 - 1215 =





First children start off by crossing them off and then progressing to exchanging, following the same process as Year 3 but with 4 digits.

## Abstract

Return to the expanded column method and use practical resources and pictures to reinforce previous learning from Year 3.

Children should then extend to subtracting four-digit numbers using this method. Ensure children have the opportunity to apply this method to a variety of different contexts, including money and measures.

$$2000 + \frac{800}{900} + \frac{20}{30} + \frac{1}{8}$$

$$- 1000 + 600 + 40 + 9$$

$$1000 + 200 + 90 + 9$$

The children can then progress to using the more formal column subtraction.

Ensure children have experience of using this method for subtraction where there is a 0 in the column they need to exchange from.

# Concrete

# Pictorial

# Abstract

Children should be increasingly confident with column subtraction and may only need to use practical resources to recap their memory of the exchanging.



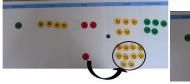
Children should then be introduced to subtracting including decimals in the context of money and measure, adding the zeros into place value columns that are empty.

$$143.6 - 31.75 =$$





They need to realise that there are no hundredths to subtract so they need to exchange 1 tenth for 10 hundredths.





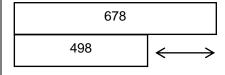
They then need to spot that there are not enough tenths to take away 7 tenths. Therefore, they need to exchange 1 whole one for 10 tenths.

Children can begin to show the calculations using pictorial representations. However, they should be beginnining to realise that this is not the most efficient way of solving the problem.

When confident, children can find their own way to record the exchange/regrouping.

Children should use the bar model to help them to solve complex word problems involving subtraction.

e.g
There are 678 children in a school. On Monday, only 498 attend school. How many children were not in school?



Children's confidence should be growing with subtraction and they should use the column method to subtract increasingly larger and complex numbers, in a range of contexts.

$$635304$$

$$-421785$$

$$213519$$

Children will then be able to use this method to subtract numbers up to 1, 000, 000 where there are zeros in some place value columns.

Again, the children should ensure that the decimal places are in line and that other numbers line up in their correct place value columns. Here a place holder has been added to remind children that they need to exchnage from the ones column.

# Concrete

# Pictorial

Year 6

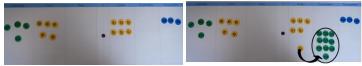
# Abstract

When introducing the millions column to the children in subtraction questions, it may be beneficial to show them using place value counters.

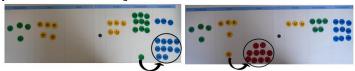


Children will need a lot of practice with subtracting decimals and understanding what to do when there are different amounts of decimals in the numbers that they are subtracting. Using practical equipment as shown below encourages them to place a 0 in these columns.

450.603 - 27.244 =



You can't subtract 4 thousandths from 3 thousandths without going into a decimal answer. There are no tenths to exchange so they need to exchange 1 tenth for 10 hundredths first.



They can then take away the thousandths, hundredths and tenths. However, there are no ones in the column so they need to exchange 1 ten for ten ones to complete the subtraction.

As in Year 5, children can begin by showing the calculations using pictorial representations.

However, they should beginning to realise that this is not the most efficient way of solving the problem.

They should be able to use the bar model to solve problems that require multi-steps of both addition and subtraction. They should be able to use this to help them to solve problems where they need to find how much is left.

e.g
England recruited 82,016 soldiers
over a year, trained 5, 399
nurses and 4, 375 pilots. What is
the difference in the amount of
soldiers compared with the
amount of pilots and nurses
combined?

82,016			
5,399	14,375	<b>←</b>	$\rightarrow$

Children will be expected to calculate with numbers that have numbers in the millions column where there are multiple Os.

$$76\overset{?}{3}\overset{?}{0}\overset{?}{3}\overset{?}{0}\overset{?}{0}$$

$$-421785$$

$$7208515$$

They should also use the compact column method to subtract money and measures, including decimals with 3 or more decimal places. They should ensure that all decimals are placed in the correct columns.

Children can fill empty decimal places with zeros to show the place value in each column. Shown here in red.

# Multiplication



# Concrete

In Year 3 children should begin with using cubes to represent the multiplication tables in arrays. They should be working with digits in the times tables that they are familiar with: 1, 2, 3, 4, 5 and 8.

They should explore the different ways of partitioning the multiplication tables.



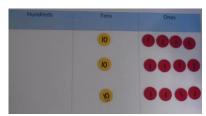




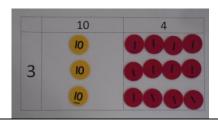
 $3 \times 4 = 12$   $3 \times 2 + 3 \times 2 = 12$ 

Once they have fully explored the times tables, children can move on to multiplying 2 digit numbers larger than 12 using place value coutners.

$$13 \times 3 =$$

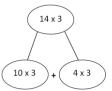


When they are confident with this they can begin to explore the grid method with the support of arrays. E.g. "We need 3 rows of 10 and 3 rows of 4".



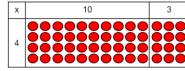
# Pictorial

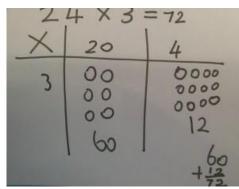
When partitioning the multiplication tables, children should be encouraged to use the part whole model and bar model as different representations.



14 x 3	
10 x 3	4 x 3

Alongside the concrete resources, children can then begin to record using the pictorial representation. They can draw the counters, using colours to show different amounts or just use circles in the different columns to show their thinking as shown below.



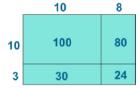


## Abstract

Once the children have a secure understanding of the above steps, the grid method can be introduced alongside a pictorial representation to start with and then the children practise and use this in a variety of different contexts.

Start with multiplying by one digit numbers and showing the clear addition alongside the grid. Moving forward, multiply by a 2 digit number showing the different rows within the grid method.

×	30	5
7	210	35

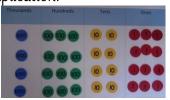


For those children who show a secure understanding of the previous steps and can use these in a variety of contexts, they may be shown how to record this as a short multiplication method using the expanded version. This should be done alongside the grid method so that children are clear on the link between the two.

Year 4				
Concrete	Pictorial	Abstract		
Children in Year should begin by exploring arrays for the multiplication tables they are yet to explore: 6, 7, 8, 9, 11 and 12. All questions	Again, the children should be confident representing their multiplication with the partwhole model and the bar model to show	First with the abstract, children will use the grid method multiplying 3-digit numbers by 1 digit numbers.		
should be based around multiplying with 6, 7, 8	different ways that it can be partitioned.	x 200 60 7		
and 9 to help them to practise their knowledge.	123 x 3	8 1600 480 56		
They should then recap the methods used in the previous year. This should then lead on to the children multiplying 3-digit numbers, beginning with numbers that do not require exchanging and moving on to numbers that do require exchanging.  e.g 123 x 3 =	Children should then use pictorial representations alongside the concrete at first. They should draw the place value counters into the grid.	1600 + 480 + 56 = 2136  They should then be shown the formal written method of short multiplication initially with no exchanging and the expanded method. Then progressing on to where exchanges are required.		
124 x 4 =    Marketon   Tens   Oues   Oues	Pictorial representation of grid method.	$\begin{array}{c} 123 \\ \underline{X \ 3} \\ 9 \ (3 \times 3) \\ \underline{60 \ (20 \times 3)} \\ \underline{+300 \ (100 \times 3)} \\ 369 \\ \end{array}$ $\begin{array}{c} \underline{+300 \ (100 \times 3)} \\ 372 \\ \end{array}$ Once the children are confident with this they can be moved on to the compact version of short multiplication. They should do this alongside the grid method to aid understanding. $\begin{array}{c} 124 \\ \underline{\times 3} \\ 372 \\ \end{array}$ The children should record their exchanges above the calculation to prevent confusion when moving on to long multiplication.		

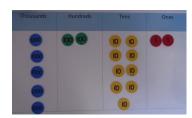
# Concrete

Children should recap learning from the previous year. They shluld then use practical equipment to multiply 4-digit numbers using short multiplication.



The children will need to exchange 10 ones for 1 ten and then exchange 10 hundreds for 1 thousand.





# Pictorial

36 x 25	
36 x 20	36 x 5

Children should be able to use the bar model to represent calculations that involve multiplying two digit numbers by 2-digit numbers.

The children can choose to draw counters to support them when multiplying using short multiplication. However, they should be able to quickly move on to using the abstract method.

Any child who is still not ready for the formal written method will continue to use grid multiplication.

# Abstract

Children start by multiplying 4 digits by 1 digit using the expanded method of short multiplication. They'll then move onto calculating these using the compact method of short multiplication.

Children should also be introduced into the formal method of long multiplication. They will need to be reminded about lining up their numbers clearly in columns, ensuring they are aware of the place value that they are multiplying. They should be able to see the calculation in 2 different parts.

$$36 \times 25 = 36 \times 20 + 36 \times 5$$

They should explore these separately before putting them together. To begin with they will expand the calculation. If it helps, children can write out what they are solving next to their answer. This moves to the more compact method.

#### Year 6 **Pictorial Abstract** Concrete Children will again recap the use of place Children should be able to use the bar As in Year 5, children will use short multiplication when value counters when using short multiplication model to represent calculations that multiplying by 1 digit numbers. They should then progress to as in previous years with increasing amounts involve multiplying 4-digit numbers by multiplying using this method involving decimals. It is vital that of exchanging. 2-digit numbers. the children line up the decimal places. 1236 x 25 $4\overset{1}{1}3.\overset{1}{2}4$ 1236 x 5 1236 x 20 The children can choose to draw 1652.96 The children will need to exchange 10 ones for counters to support them when 1 ten and then exchange 10 hundreds for 1 Children should become increasingly confident with long multiplying using short multiplication. thousand. multiplication and multplying up to 4 digit numbers by 2-digit However, they should be able to numbers. quickly move on to using the abstract method. 4436 Any child who is still not ready for the Children must remember to formal written method will continue to 25 include the place holder use grid multiplication. shown here in red (a 22180 common error). 88720

110900

# Division



Children will use arrays to work out division facts. They should then be able to turn the array around to write the other division sentence that can be derived from this array.





$$24 \div 4 = 6$$

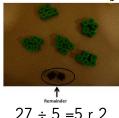
$$24 \div 6 = 4$$

The children should also know the difference between grouping and sharing questions.



 $25 \div 5 = 5$ 

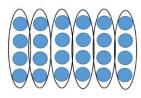
Then use practical resources to find remainders.



Finally, use place value counters to explore this.

Place value dividing image

The children will use the pictorial representations alongside the concrete. They will draw the counters and then group them without remainders at first.



They can also draw dots and group them to divide an amount and clearly show a remainder. This will help them to ccomplete written divisions and show the remainder of those that cannot be grouped using r.









The children could also use the bar model to represent division, using visual representations of the counters.

		2	4		
0000	••••	••••	••••	••••	••••

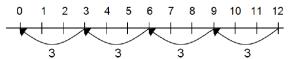
Beginning with exploring division with their tables knowledge, children will complete division questions that require them to use the inverse operation.

e.q

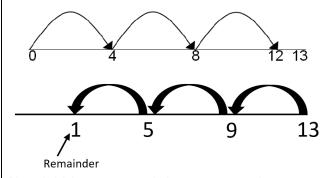
If a child knows that  $3 \times 4 = 12$ , they can use this to answer:

 $12 \div 4 = 3$  and  $12 \div 3 = 4$ 

The children should use the number line to make jumps in division questions without remainders.



Children also continue to develop their understanding of using repeated addition and repeated subtraction on a number line to divide, and also to find remainders.



If a child has mastered division using these methods, formal methods will be introduced (short division).

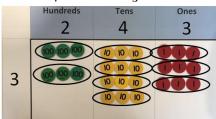
# Concrete

# Pictorial

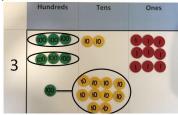
# **Abstract**

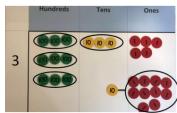
Children should use practical and pictorial representations to ensure that they are confident with the difference between grouping and sharing.

Children will begin to divide using place value counters. First of all they will divide simple calculations where there are no remainders or exchanging. They should put rings around the groups and can use the place value grid as a division grid.



The children will then be able to move onto dividing when exchanges are needed. They should be able to exchange from all place value columns.

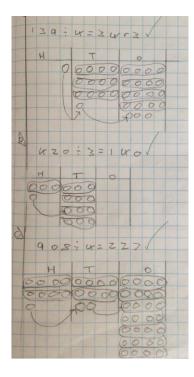




The children should then use practical resources to explore what it means when there is an answer with a remainder.



The children should then be encouraged to draw the pictorial representation alongside the concrete. Again following the process of exchanging and remainders.



Once children are secure with division as grouping and sharing, using number lines, arrays etc. short division for larger two-digit numbers can be introduced. To start with, this should be introduced with numbers that have no remainders within, or at the end of the calculation. Children should be introduced to the correct terminology for division questions.

$$936 \div 3 = 312$$

- The dividend is 936 (the number being divided)
- The divisor is 3 (the number you are dividing by)
- The quotient is 312 (the answer)

Following the same process alongside the pictorial, the children will divide without exchanges and then move onto exchanges.

They will then begin to explore remainders.

$$\begin{array}{r}
2 4 5 r 1 \\
3 7^{1}4^{2}6
\end{array}$$

	Year 5	
Concrete	Pictorial	Abstract
The children should begin by dividing 3 and 4 digit numbers using practical resources where exchanging is needed as they should be confident with this from the previous year.  8547 ÷ 7 =  They should then move on to dividing to find remainders using this approach with exchanging inside the calculation. The remainder will be left over at the end.  16531 ÷ 4 =  It is important to explore what it means if the remainder is larger than the divisor. This will help them in long division in Year 6.	The children can then progress to recording this by drawing counters alongside the practical resources.	Children should recap their previous work on short division- at this stage with no final remainder, but with remainders which need carrying within the calculation. This will now be extended to include four-digit numbers divided by a one-digit number. $\frac{1058}{952^{7}2}$ They should then progress to dividing where there is a remainder. $\frac{0938}{46^{1}9^{4}2}$ Children should be confident with dividing by numbers up to 12, applying their knowledge of their times table facts. They should also have regular practice where there are 0s involved. $\frac{0708}{1285^{1}0^{10}2}$

# Concrete

Children will begin by recapping short division from previous year. They will then progress to dividing decimals using this method.

 $3606.96 \div 3 = 1202.32$ 



The children should then progress to dividing whole numbers to find decimal answers.



The one will need to be exchanged for 10 tenths and then the extra 2 tenths for 20 hundredths. .



They continue working through the decimal places until there is no remainder.



The children will then have their answer.

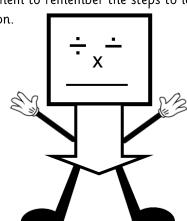
# Pictorial

With deep understanding being built upon from previous years, it is unlikely that the chidren will need practical resources and pictorial representations.

When calculating with decimals, children can show the division using pictorial representations of the place value counters to support their learning. This should be used alongside the concrete.

They can use this alongside the method of long divison. However, it should be explored that this may not be the most efficient use of time.

The children like to use the visual representation of the 'Division Man' to the help them to remember the steps to long division.



# **Abstract**

When dividing to find remainders, the children should be able to record these in different forms. E.g.

With a remainder:

$$\frac{1904}{59^452^2}$$

As a fraction:

$$\frac{1904}{5} \cdot \frac{2^{2}}{5}$$

As a decimal:

When the children are confident with this method, they will then move onto long division.

J	017
Step 1:	25 425
Divide	oŢĬ
Step 2:	
Multiply	42
Step 3:	<u>25</u> ↓
Subtract	175
Step 4:	175
Bring down	000

Help Box
1 x 25 = 25
2 x 25 = 50
5 x 25 = 125
10 x 25 = 250

To help them, the children can write known facts in a help box to support them with their division.